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2 **Education Sector Assessment Using Linguistic Quadripartitioned Single-Valued**
3 **Neutrosophic Soft Sets**

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11 **Abstract**

12 Recent time uncertainty analysis and its characterization become more complex due to
13 dark data sets. It became more crucial in the case of the education sector where large
14 number of dark data set generated related to measuring student performance. This
15 becomes more complex while dealing with linguistic information and its significance in
16 case of student performance measurement. One of the reason is that these types of data
17 is based on human quantum Turiyam consciousness and some time in unconsciousness
18 way. To deal with these types of dark data generated in unconscious way this paper try
19 to introduce the linguistic Quadripartitioned single-valued neutrosophic set (LQSVNS).
20 It defines each object of the universe by four independent linguistic variables known as
21 truth, contradiction, unknown, and false linguistic variables. Same time some operations
22 and properties based on LQSVNNs are extensively studied in this paper using a
23 combination of soft set (SS) known as the linguistic Quadripartitioned single-valued

24 neutrosophic soft set (LQSVNSS). Moreover, a distance similarity measured-based
25 model under the LQSVNSSs is investigated with an illustrative example of the
26 education sector.

27 **Keywords:** Big data; Four Valued Data; Linguistic Quadripartitioned single-valued
28 neutrosophic set; Quadripartitioned single-valued neutrosophic set; Turiyam set

29 1. Introduction

30 The last decade most of the researchers paid attention towards computing with words
31 and its vagueness. To deal with this problem the algebra of Fuzzy Set (FS) (Zadeh,
32 1965) is introduced with the aid of the truth-grade function (μ) and its membership value
33 defined in [0,1]. Atanassov et al. (Atanassov & Stoeva, 1986) initiated an intuitionistic
34 fuzzy set (IFS) to represent the acceptance and the rejection part via dependent true and
35 false membership values μ and ν respectively via inequality $0 \leq \mu + \nu \leq 1$. The problem
36 arises with IFS also in case $\mu + \nu > 1$ or they are independent of each other. To solve this
37 issue, Pythagorean fuzzy sets (PFSs) (Yager, 2013), q-rung orthopair fuzzy sets (q-
38 ROFSs) (Yager, 2016) are introduced. In a PFS and a q-ROFS the true (μ) and false (ν)
39 membership grades of a particular element are restricted with conditions $0 \leq \mu^2 + \nu^2 \leq 1$
40 and $0 \leq \mu^q + \nu^q \leq 1$ respectively with its various applications (Garg, 2016; Thao, 2020;
41 Xiao & Ding, 2019; Ejegwa, 2020).

42 The problem arises when the hesitant part of IFS become independent of true and false
43 membership values. To deal with this type of indeterminacy Single-valued
44 Neutrosophic set (SVNS) introduced Smarandache (Smarandache, 2005; Wang,
45 Smarandache, Zhang & Sunderraman, 2010). It is applied in various fields (Broumi et
46 al. 2023; Caballero & Broumi 2023; Sivasankar & Broumi 2023). The other way to

47 make a decision through the application of NST. One is to use neutrosophic rough sets
48 (Broumi et al., 2014; Salama & Broumi, 2014) and the other is to use aggregation
49 operators. To extend neutrosophic set to MCDA problems, Bao and Yang (2017)
50 proposed a model integrating single valued neutrosophic refined sets with rough sets
51 while Bo et al. (2018) utilized multi-granulation neutrosophic rough sets. Moreover,
52 many mixed models with NST have been proposed, such as n-dimension single valued
53 neutrosophic refined rough sets (Yang et al., 2017; Zhao & Zhang, 2018a) and hesitant
54 neutrosophic rough sets (Zhao & Zhang, 2018b, 2020). In addition, there is a combined
55 ITARA with TOPSIS-AL approach based neutrosophic sets for risk assessment of
56 university sustainability (Lin & Lo, 2023). The problem arises while dealing with
57 contradictory, unknown, ambiguous, or computing its complement as discussed by
58 Belnap (Belnap Jr, 1977). It can be useful to encounter contradictory facts, human super
59 consciousness, and solve various machine intelligence problems based on human
60 cognitive intelligence. Due to this, the SVNS is extended as Quadripartitioned single-
61 valued neutrosophic set (QSVNS) (Chatterjee et al. 2016 a, b). It can represent the
62 dependent data set and its membership via n-valued refined neutrosophic set
63 (Smarandache, 2013) which give several authors to study QSVNS (Roy, Lee, Pal &
64 Samanta, 2020; Mohanasundari & Mohana, 2020; Mohan & Krishnaswamy, 2020a;
65 Sinha & Majumdar, 2020) and its hybrid model (Mohan & Krishnaswamy, 2020b;
66 Kamaci, 2021; Sinha, Majumdar & Broumi, 2022). It becomes more useful when soft
67 set (SS) (Molodtsov, 1999) theory is connected with this logic. The reason is soft set
68 offers a more general framework to present uncertainty without any restriction. The
69 hybridization with soft set utilizes in many areas such as decision-making problems
70 (Maji, Roy & Biswas, 2002; Kong, Wang & Wu, 2011), medical diagnosis

71 (Muthukumar & Krishnan, 2016; Xiao, 2018), etc. Moreover, a combination of SS and
72 QSVNS provides a new theoretical concept known as quadripartitioned single-valued
73 neutrosophic soft set (QSVNSS) that can be viewed as a special type of
74 quadripartitioned neutrosophic soft set (QNSS) (Radha, Mary & Smarandache, 2021).
75 This set is utilized successfully in case of uncertainty measurement in interval-valued
76 possibility QSVNSS (Chatterjee et al. 2016b), and topological space based on QNSS
77 (Kumar & Mary, 2021). This current paper focused on Quadripartitioned single-valued
78 neutrosophic soft set (LQSVNSS) for dealing with the qualitative data.

79 However, dealing with qualitative data required human consciousness for the precise
80 representation of contradictory events. Sometimes it can be represented via linguistic
81 variables in the case of known objects or dependent variables. For example, small, very
82 small, almost small, not small, quite small, not very small, etc. The linguistic setting is a
83 very popular and interesting topic to measure the uncertainty that arises due to human
84 thoughts. In the research article (Zadeh, 1975), Zadeh first introduced the concept of
85 linguistic variables in approximate reasoning. According to him, a linguistic variable is
86 characterized by a quintuple $(\mathfrak{S}, \mathfrak{T}(\mathfrak{S}), \mathfrak{U}, \mathfrak{G}, \mathfrak{M})$, where \mathfrak{S} is a variable, $\mathfrak{T}(\mathfrak{S})$ is the term set
87 of \mathfrak{S} , \mathfrak{U} is a set of the universe, \mathfrak{G} is a rule that generates $\mathfrak{T}(\mathfrak{S})$, and \mathfrak{M} is a semantic rule
88 associated with a linguistic value. Liu et al. (Liu & Zhang, 2010) utilized the risk-based
89 linguistic variable in MADM. Herrera et al. (Herrera & Herrera-Viedma, 2000)
90 presented a linguistic decision analysis to solve the decision problem. Xu (Xu, 2004)
91 proposed the linguistic aggregate operators for group decision-making. A fuzzy set-
92 based linguistic value is proposed in (Dohnal, 1983; Bonissone, 1980). Zhang (Zhang,
93 2014) introduced the linguistic intuitionistic fuzzy set (LIFS) that is characterized by
94 linguistic membership and non-membership degrees. Garg et al. (Garg & Kumar, 2018)

95 presented some aggregate operators on LIFSs in GDM. He also presented the linguistic
96 Pythagorean fuzzy set (LPFS) (Garg, 2018) to address uncertain linguistic information
97 in a better way. Also, q-rung orthopair fuzzy sets are based on linguistic information
98 studied in (Lin, Li & Chen, 2020; Akram, Naz, Edalatpanah & Mehreen, 2021; Liu &
99 Liu, 2019; Wang, Ju & Liu, 2019). The fusion of neutrosophic set and its hybrids with
100 linguistic set resulted in linguistic neutrosophic sets (LNSs) in MCDM (Li, Zhang &
101 Wang, 2017), interval complex neutrosophic sets under linguistic information (Dat,
102 Thong, Ali, Smarandache, Abdel-Basset & Long, 2019), multi-objective linguistic
103 neutrosophic matrix games (Bhaumik, Roy & Weber, 2021). Kamaci (Kamaci, 2021)
104 initiate the linguistic single-valued neutrosophic soft set (LSVNS) and proposed a game
105 theory model via the TOPSIS technique.

106 Some other methods are proposed for dealing the human cognition (Singh, 2021) in
107 case of unknown or undefined impossible objects using human quantum Turiyam
108 cognition (Singh, 2022a). It represents the qualitative data based on four dimensions
109 whereas the fourth dimension is represented by Human Turiyam consciousness. It is
110 independent of true, false, uncertain, or contradictory membership values of the given
111 event (Singh, 2022b). It is based on time-based measurement, human
112 superconsciousness, or quantum cognition distinct from any of the available set as
113 discussed in Singh (2023c). **It represents the error measured in dark data sets due to**
114 **human consciousness rather than unconsciousness value of Quadripartitioned set.** It can
115 be represented via the Turiyam matrix (Ani, Mashadi & Gemawati, 2023) for precise
116 analysis of knowledge-processing tasks based on the Turiyam relation (Ganati,
117 Srinivasa Rao Repalle & Ashebo, 2023) and its graph (Ganati, Srinivasa, Ashebo &
118 Amini, 2023). It gives a way to represent the data set based on human quantum

119 cognition (Singh, 2023a,b,c) for dealing the self-driving cars (Said et al., 2022), robotics
120 (Silva, 2022) or mathematical exploration in school teaching (Singh 2023b). The
121 problem arises when data sets and their uncertainty are represented without human
122 cognition beyond the three polar spaces (Singh, 2023c; Siraj, Naeem & Said, 2023;
123 Naeem & Divvaz, 2023; Naeem, Riaz & Karaaslan, 2021; Naeem, Riaz & Afzal, 2019;
124 Siraj, Fatima, Afzal, Naeem & Karaaslan, 2022). It requires a new set theory for dealing
125 the qualitative data sets contain static uncertainty rather than involvement of human
126 consciousness (Singh 2023b). To achieve this goal, the current paper focused on dealing
127 with uncertainty in known qualitative data and its refinement using linguistic single-
128 valued neutrosophic soft set (LSVNS) rather than human consciousness.

129 Table 1 gives an overview and distinction among each of the available approaches.

130 Other parts of the paper are organized as follows: Section 2 provides a brief background
131 about the mathematical concepts. Section 3 provides the basis of weighted aggregation
132 for the proposed method and its illustration. In Section 4, LQSVNSSs and their
133 properties are executed. Application of the present study is briefly discussed in Section
134 5. Section 6 contains the conclusion followed by acknowledgment and references.

135 **2. Basic Mathematical Concepts**

136 In this section, some basic concepts are reviewed to ease the discussion in the
137 subsequent sections:

138 **2.1 Linguistic Term Set (LTS)**

139 **Definition 2.1** (Zadeh, 1975) Let $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ be a finite linguistic term set with
 140 cardinality $p + 1$ where p is a positive integer. Also, we consider s_m as a possible value of
 141 a linguistic variable. Then the LTS must satisfy the following properties:

142 (i) $s_m \geq s_n$ if $m \geq n$ and $s_m \leq s_n$ if $m \leq n$ (Order relation)

143 (ii) $neg(s_m) = s_n$ where $m + n = p$ (negation)

144 This concept is based on discrete LTS. To extend this concept to continuous LTS, see
 145 the next definition.

146 **Definition 2.2** (Xu, 2004) Let $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ be a finite linguistic term set with
 147 cardinality $p + 1$ where p is a positive integer. Then, $\mathfrak{S}_{[0,p]} = \{s_m : s_0 \leq s_m \leq s_p, m \in [0,p]\}$ is said
 148 to be a continuous LTS for \mathfrak{S} .

149 2.2 Single-Valued Neutrosophic Set (SVNS)

150 **Definition 2.3** (Smarandache, 2005) A single-valued neutrosophic set (SVNS) \mathcal{H} over \mathcal{L}
 151 is defined as

152 $\mathcal{H} = \{(\ell_i, \langle \mu_{\mathcal{H}(\ell_i)}, \vartheta_{\mathcal{H}(\ell_i)}, \nu_{\mathcal{H}(\ell_i)} \rangle) : \ell_i \in \mathcal{L}\}$ where $\mu_{\mathcal{H}(\ell_i)}, \vartheta_{\mathcal{H}(\ell_i)}, \nu_{\mathcal{H}(\ell_i)} \in [0,1]$ represent the degrees
 153 of truth, indeterminacy, and falsity memberships respectively
 154 with $0 \leq \mu_{\mathcal{H}(\ell_i)} + \vartheta_{\mathcal{H}(\ell_i)} + \nu_{\mathcal{H}(\ell_i)} \leq 3$.

155 2.3 Linguistic Single-Valued Neutrosophic Set (LSVNS)

156 **Definition 2.4** (Li, Zhang & Wang, 2017) Let $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ be a finite linguistic
 157 term set with cardinality $p + 1$ where p is a positive integer
 158 and $\mathfrak{S}_{[0,p]} = \{s_m : s_0 \leq s_m \leq s_p, m \in [0,p]\}$. Then a LSVNS \mathcal{G} in \mathcal{L} is defined as

159 $\mathcal{G} = \{(\ell_i, \langle \mathcal{G}_{\mathcal{Z}_\mu}(\ell_i), \mathcal{G}_{\mathcal{Z}_\theta}(\ell_i), \mathcal{G}_{\mathcal{Z}_\nu}(\ell_i) \rangle) : \ell_i \in \mathcal{L}\}$ where $\mathcal{G}_{\mathcal{Z}_\mu}(\ell_i), \mathcal{G}_{\mathcal{Z}_\theta}(\ell_i), \mathcal{G}_{\mathcal{Z}_\nu}(\ell_i) \in \mathbb{S}_{[0,p]}$ denote
 160 the linguistic truth, indeterminacy, and falsity degrees of $\ell_i \in \mathcal{L}$ respectively such that
 161 $0 \leq \mu, \theta, \nu \leq p$ and $0 \leq \mu + \theta + \nu \leq 3p$.

162 2.4 Quadripartitioned Single-Valued Neutrosophic Set (QSVNSS)

163 **Definition 2.5** (Chatterjee et al. 2016 a,b) A QSVNS \mathcal{Q} in \mathcal{L} is defined
 164 as $\mathcal{Q} = \{(\ell_i, \langle \mu_{\mathcal{Q}}(\ell_i), \xi_{\mathcal{Q}}(\ell_i), \zeta_{\mathcal{H}}(\ell_i), \nu_{\mathcal{Q}}(\ell_i) \rangle) : \ell_i \in \mathcal{L}\}$ where $\mu_{\mathcal{Q}}(\ell_i), \xi_{\mathcal{Q}}(\ell_i), \zeta_{\mathcal{H}}(\ell_i), \nu_{\mathcal{Q}}(\ell_i) \in [0,1]$ denote the
 165 truth, contradiction, unknown, and falsity membership grades respectively
 166 with $0 \leq \mu_{\mathcal{Q}}(\ell_i) + \xi_{\mathcal{Q}}(\ell_i) + \zeta_{\mathcal{H}}(\ell_i) + \nu_{\mathcal{Q}}(\ell_i) \leq 4$. It is distinct from Turiyam where each parameter is
 167 independent and the last dimension is based on human Turiyam consciousness. It means
 168 Turiyam contains two truth values rather than one in QSVNSS. Same time the Liberal
 169 values in Turiyam are independent from true false or uncertainty whereas in
 170 Quadripartitioned Single-Valued Neutrosophic not the same. In this case, this set theory
 171 is helpful in some cases where human quantum cognition is not required. We required
 172 membership value in a partition way for dealing with the contradiction among the
 173 opinion.

174 2.5 Quadripartitioned Single-Valued Neutrosophic Soft Set (QSVNSS)

175 **Definition 2.6** (Radha, Mary & Smarandache, 2021) Let \mathcal{L} be an initial universe and \mathcal{E}
 176 be a set of parameters where $\mathcal{A}(\neq \varphi) \subseteq \mathcal{E}$. Let $\wp(\mathcal{L})$ signifies the collection of all
 177 Quadripartitioned single-valued neutrosophic sets of \mathcal{L} . Then the pair $(\mathcal{F}, \mathcal{A})$ is considered
 178 as a QSVNSS over \mathcal{L} where $\mathcal{F}: \mathcal{A} \rightarrow \wp(\mathcal{L})$.

179 3. Linguistic Quadripartitioned Single-Valued Neutrosophic Set (LQSVNS)

180 In this section dealing of linguistic variables which are qualitative is discussed via
 181 LQSVNS. Same time different aggregate operators and distance measures associated
 182 with LQSVNS is also introduced in this section for dealing the linguistic static
 183 uncertainty arises unconsciously.

184 **Definition 3.1** Let $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ be a continuous finite linguistic term set with

185 cardinality $p + 1$ where p is a positive integer and $\mathfrak{S}_{[0,p]} = \{s_m : s_0 \leq s_m \leq s_p, m \in [0, p]\}$.

186 Then a LQSVNS δ in \mathcal{L} is defined as

187 $\delta = \{(\ell_i, \langle \delta_{s_\mu}(\ell_i), \delta_{s_\xi}(\ell_i), \delta_{s_\zeta}(\ell_i), \delta_{s_\nu}(\ell_i) \rangle) : \ell_i \in \mathcal{L}\}$ where $\delta_{s_\mu}(\ell_i), \delta_{s_\xi}(\ell_i), \delta_{s_\zeta}(\ell_i), \delta_{s_\nu}(\ell_i)$

188 $\in \mathfrak{S}_{[0,p]}$ denote the linguistic truth, contradiction, uncertain, and falsity degrees of $\ell_i \in \mathcal{L}$

189 respectively such that $0 \leq \mu, \xi, \zeta, \nu \leq p$ and $0 \leq \mu + \xi + \zeta + \nu \leq 4p$.

190 In short, $\delta = \langle \delta_{s_\mu}, \delta_{s_\xi}, \delta_{s_\zeta}, \delta_{s_\nu} \rangle$ denote the linguistic Quadripartitioned single-valued

191 neutrosophic number (LQSVNN). Moreover, the set of all LQSVNNs on \mathfrak{S} is denoted

192 by $\mathfrak{N}_{[0,p]} = \{ \langle \delta_{s_\mu}, \delta_{s_\xi}, \delta_{s_\zeta}, \delta_{s_\nu} \rangle : \delta_{s_\mu}, \delta_{s_\xi}, \delta_{s_\zeta}, \delta_{s_\nu} \in \mathfrak{S}_{[0,p]} \}$.

193 **Example 3.2** Let $\mathcal{L} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ be a set of alternatives and

194 $\mathfrak{S} =$
 195 $\{s_0 = \text{very sensitive (VS)}, s_1 = \text{quite sensitive (QS)}, s_2 = \text{high sensitive (HS)}, s_3 = \text{very high sensitive (VHS)}, s_4 = \text{medium (M)}, s_5 = \text{low (L)}$
 $\}$ be a LTS.

196 If we consider the following

197 $\delta = \{(\ell_1, \langle s_1, s_2, s_0, s_1 \rangle), (\ell_2, \langle s_0, s_0, s_4, s_1 \rangle), (\ell_3, \langle s_1, s_0, s_1, s_3 \rangle), (\ell_4, \langle s_2, s_3, s_3, s_4 \rangle),$
 198 $\ell_5, \langle s_5, s_0, s_3, s_3 \rangle)\}$

199 Then δ denotes the LQSVNS over \mathcal{L} .

200 **Definition 3.3** Let $\delta = \langle \delta_{\xi_{\mu}}, \delta_{\xi_{\zeta}}, \delta_{\xi_{\nu}}, \delta_{\xi_{\nu}} \rangle \in \mathbb{N}_{[0, p]}$ be a LQSVNN. Then the score function

201 associated to δ is defined as

$$202 \quad U(\delta) = \frac{\delta_{\xi_{\mu}} + \delta_{\xi_{\zeta}} - \delta_{\xi_{\nu}} - \delta_{\xi_{\nu}}}{2} \text{ Or } \frac{\mu + \zeta - \nu}{2} \in [-p, p] \quad (1)$$

203 **Definition 3.4** Let $\delta = \langle \delta_{\xi_{\mu}}, \delta_{\xi_{\zeta}}, \delta_{\xi_{\nu}}, \delta_{\xi_{\nu}} \rangle \in \mathbb{N}_{[0, p]}$ be a LQSVNN. Then the accuracy

204 function associated to δ is defined as

$$205 \quad \mathfrak{H}(\delta) = \frac{\delta_{\xi_{\mu}} + \delta_{\xi_{\zeta}} + \delta_{\xi_{\nu}} + \delta_{\xi_{\nu}}}{4} \text{ Or } \frac{\mu + \zeta + \nu}{4} \in [0, p] \quad (2)$$

206 **Definition 3.5** Let $\delta_1 = \langle \delta_{\xi_{\mu_1}}, \delta_{\xi_{\zeta_1}}, \delta_{\xi_{\nu_1}}, \delta_{\xi_{\nu_1}} \rangle$ and $\delta_2 = \langle \delta_{\xi_{\mu_2}}, \delta_{\xi_{\zeta_2}}, \delta_{\xi_{\nu_2}}, \delta_{\xi_{\nu_2}} \rangle$ be two

207 LQSVNNs. Based on the score and accuracy function (see Definition 3.3 & 3.4), to

208 compare these two LQSVNNs, we define the following:

209 a) if $U(\delta_1) > U(\delta_2)$ then $\delta_1 > \delta_2$ i.e. δ_1 is more preferable than δ_2

210 b) if $U(\delta_1) = U(\delta_2)$ then

211 i) for $\mathfrak{H}(\delta_1) = \mathfrak{H}(\delta_2)$, $\delta_1 = \delta_2$

212 ii) for $\mathfrak{H}(\delta_1) > \mathfrak{H}(\delta_2)$, $\delta_1 > \delta_2$

213 **Example 3.6** Let $\mathfrak{S} = \{s_m : s_0 \leq s_m \leq s_7, m \in [0, 7]\}$ be a LTS. Also, we consider

214 $\delta_1 = \langle s_1, s_3, s_4, s_5 \rangle$, $\delta_2 = \langle s_0, s_4, s_2, s_3 \rangle$, $\delta_3 = \langle s_3, s_5, s_4, s_2 \rangle$, and $\delta_4 = \langle s_7, s_6, s_2, s_3 \rangle$ be LQSVNNs

215 derived from \mathfrak{S} . By using equation (1), we obtain the following

$$216 \quad U(\delta_1) = \frac{1+3-4-5}{2} = -2.5, \quad U(\delta_2) = \frac{0+4-2-3}{2} = -0.5, \quad U(\delta_3) = \frac{3+5-4-2}{2} = 1, \quad U(\delta_4) = \frac{7+6-2-3}{2} = 4$$

217 Thus, we rank the numbers as $\delta_4 > \delta_3 > \delta_2 > \delta_1$

218 **3.1 Operational Laws on LQSVNNs**

219 **Definition 3.7** Let $\delta_1 = \langle \delta_{\varepsilon_{\mu'_1}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_1}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}, \delta_{\varepsilon_{\nu'_1}} \rangle$ and $\delta_2 = \langle \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}, \delta_{\varepsilon_{\nu'_2}} \rangle$ be two

220 LQSVNNs. Then we have the following laws:

221 a) $\delta_1 \cup \delta_2 = \langle \max(\delta_{\varepsilon_{\mu'_1}}, \delta_{\varepsilon_{\mu'_2}}), \max(\delta_{\varepsilon_{\xi'_{\varepsilon'_1}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}}), \min(\delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}}), \min(\delta_{\varepsilon_{\nu'_1}}, \delta_{\varepsilon_{\nu'_2}}) \rangle$

222 $= \langle \delta_{\max(\varepsilon_{\mu_1}, \varepsilon_{\mu_2})}, \delta_{\max(\varepsilon_{\xi_1}, \varepsilon_{\xi_2})}, \delta_{\min(\varepsilon_{\zeta_1}, \varepsilon_{\zeta_2})}, \delta_{\min(\varepsilon_{\nu_1}, \varepsilon_{\nu_2})} \rangle$

223 b) $\delta_1 \cap \delta_2 = \langle \min(\delta_{\varepsilon_{\mu'_1}}, \delta_{\varepsilon_{\mu'_2}}), \min(\delta_{\varepsilon_{\xi'_{\varepsilon'_1}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}}), \max(\delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}}), \max(\delta_{\varepsilon_{\nu'_1}}, \delta_{\varepsilon_{\nu'_2}}) \rangle$

224 $= \langle \delta_{\min(\varepsilon_{\mu_1}, \varepsilon_{\mu_2})}, \delta_{\min(\varepsilon_{\xi_1}, \varepsilon_{\xi_2})}, \delta_{\max(\varepsilon_{\zeta_1}, \varepsilon_{\zeta_2})}, \delta_{\max(\varepsilon_{\nu_1}, \varepsilon_{\nu_2})} \rangle$

225 c) $\delta_1 \supseteq \delta_2 \Rightarrow \delta_{\varepsilon_{\mu'_1}} \geq \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_1}}} \geq \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}} \leq \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}}, \delta_{\varepsilon_{\nu'_1}} \leq \delta_{\varepsilon_{\nu'_2}}$

226 d) $\delta_1 = \delta_2 \Rightarrow \delta_{\varepsilon_{\mu'_1}} = \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_1}}} = \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}} = \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}}, \delta_{\varepsilon_{\nu'_1}} = \delta_{\varepsilon_{\nu'_2}}$

227 e) $\delta_1^c = \langle \delta_{\varepsilon_{\nu'_1}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_1}}}, \delta_{\varepsilon_{\mu'_1}} \rangle$ where δ_1^c indicates the complement of δ_1 .

228 **Theorem 3.8** If $\delta_1 = \langle \delta_{\varepsilon_{\mu'_1}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_1}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_1}}, \delta_{\varepsilon_{\nu'_1}} \rangle$ and $\delta_2 = \langle \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\xi'_{\varepsilon'_2}}, \delta_{\varepsilon_{\zeta'_{\varepsilon'_2}}, \delta_{\varepsilon_{\nu'_2}} \rangle$ be in $\mathcal{N}_{[0, \rho]}$

229 then

230 1. $(\delta_1 \cup \delta_2)^c = \delta_1^c \cap \delta_2^c$

231 2. $(\delta_1 \cap \delta_2)^c = \delta_1^c \cup \delta_2^c$

232 **Proof:** Considering the Definition 3.7 (a) and (b), we can easily obtain the results.

233 **Definition 3.9** Let $\delta = \langle \delta_{s_{\mu}}, \delta_{s_{\xi}}, \delta_{s_{\zeta}}, \delta_{s_{\nu}} \rangle$, $\delta_1 = \langle \delta_{s_{\mu_1}}, \delta_{s_{\xi_1}}, \delta_{s_{\zeta_1}}, \delta_{s_{\nu_1}} \rangle$ and

234 $\delta_2 = \langle \delta_{s_{\mu_2}}, \delta_{s_{\xi_2}}, \delta_{s_{\zeta_2}}, \delta_{s_{\nu_2}} \rangle$ be three LQSVNNs in $\mathbb{N}_{[0,p]}$, and $\varrho > 0$. Then the following

235 operational laws are defined in the following:

236 a) $\delta_1 \oplus \delta_2 = \langle \delta_{s_{\mu_1} + s_{\mu_2} - \frac{s_{\mu_1} s_{\mu_2}}{p}}, \delta_{s_{\xi_1} + s_{\xi_2} - \frac{s_{\xi_1} s_{\xi_2}}{p}}, \delta_{s_{\zeta_1} + s_{\zeta_2} - \frac{s_{\zeta_1} s_{\zeta_2}}{p}}, \delta_{s_{\nu_1} + s_{\nu_2} - \frac{s_{\nu_1} s_{\nu_2}}{p}} \rangle$

237 b) $\delta_1 \otimes \delta_2 = \langle \delta_{\frac{s_{\mu_1} s_{\mu_2}}{p}}, \delta_{\frac{s_{\xi_1} s_{\xi_2}}{p}}, \delta_{s_{\zeta_1} + s_{\zeta_2} - \frac{s_{\zeta_1} s_{\zeta_2}}{p}}, \delta_{s_{\nu_1} + s_{\nu_2} - \frac{s_{\nu_1} s_{\nu_2}}{p}} \rangle$

238 c) $\varrho \delta = \langle \delta_{p-p(1-\frac{s_{\mu}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\xi}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\zeta}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\nu}}{p})^{\varrho}} \rangle$

239 d) $\delta^{\varrho} = \langle \delta_{p-p(1-\frac{s_{\mu}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\xi}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\zeta}}{p})^{\varrho}}, \delta_{p-p(1-\frac{s_{\nu}}{p})^{\varrho}} \rangle$

240 **Example 3.10**

241 Let

242 $\mathfrak{S} = \{s_0 = \text{flop}(F), s_1 = \text{average}(A), s_2 = \text{below average}(BA), s_3 = \text{above average}(AA), s_4 = \text{hit}(H), s_5 = \text{semi hit}(SMH), s_6 = \text{super hit}(SPH), s_7 = \text{blockbuster}(BBH), s_8 = \text{disaster}(D)\}$

243 be a LTS. Let $\delta_1 = \langle s_1, s_5, s_6, s_3 \rangle$ and $\delta_2 = \langle s_3, s_0, s_2, s_5 \rangle$ be two LQSVNNs obtained from

244 \mathfrak{S} and $\varrho = 0.6$. Then by using Definition 3.8, we obtain the following:

245 1. $\delta_1 \oplus \delta_2 = \langle \delta_{1+3-\frac{1 \cdot 3}{8}}, \delta_{5+0}, \delta_{\frac{6 \cdot 2}{8}}, \delta_{\frac{3 \cdot 3}{8}} \rangle = \langle \delta_{3.625}, s_5, \delta_{1.5}, \delta_{1.87} \rangle$

246 2. $\delta_1 \otimes \delta_2 = \langle \delta_{\frac{1 \cdot 3}{8}}, \delta_0, \delta_{6+2-\frac{6 \cdot 2}{8}}, \delta_{3+5-\frac{3 \cdot 5}{8}} \rangle = \langle \delta_{0.375}, s_0, \delta_{6.5}, \delta_{6.125} \rangle$

247 3. $\varrho \delta_1 = \langle \delta_{8-8(1-\frac{1}{8})^{0.6}}, \delta_{8-8(1-\frac{5}{8})^{0.6}}, \delta_{8(\frac{6}{8})^{0.6}}, \delta_{8(\frac{3}{8})^{0.6}} \rangle = \langle \delta_{0.615}, \delta_{3.55}, \delta_{6.73}, \delta_{4.441} \rangle$

248 4. $\delta_1^{\varrho} = \langle \delta_{8(\frac{1}{8})^{0.6}}, \delta_{8(\frac{5}{8})^{0.6}}, \delta_{8-8(1-\frac{2}{8})^{0.6}}, \delta_{8-8(1-\frac{5}{8})^{0.6}} \rangle = \langle \delta_{4.441}, s_0, \delta_{1.268}, \delta_{3.558} \rangle$

249 **3.2 Weighted Aggregation Operators of LQSVNNs**

250 **Definition 3.11** Let $\delta_j = \langle \delta_{\varepsilon_{\mu'_j}}, \delta_{\varepsilon_{\xi'_j}}, \delta_{\varepsilon_{\zeta'_j}}, \delta_{\varepsilon_{\nu'_j}} \rangle \in \mathbb{N}_{[0,p]}$ ($j = 1, 2, \dots, k$) be a class of
 251 LQSVNNs, and $\mathfrak{W}_j \in [0,1]$ denotes the weight of δ_j satisfying $\sum_{j=1}^k \mathfrak{W}_j = 1$. Then the
 252 linguistic Quadripartitioned single-valued weighted average aggregation
 253 (LQSVNWAA) operator is defined as

$$\begin{aligned}
 & LQSVNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) = \sum_{j=1}^k \mathfrak{W}_j \delta_j \\
 & = \langle \delta_{\varepsilon_{\mu'_j}}^{1 - \prod_{j=1}^k \left(1 - \frac{\varepsilon_{\mu'_j}}{p}\right)^{\mathfrak{W}_j}}, \delta_{\varepsilon_{\xi'_j}}^{1 - \prod_{j=1}^k \left(1 - \frac{\varepsilon_{\xi'_j}}{p}\right)^{\mathfrak{W}_j}}, \delta_{\varepsilon_{\zeta'_j}}^{1 - \prod_{j=1}^k \left(\frac{\varepsilon_{\zeta'_j}}{p}\right)^{\mathfrak{W}_j}}, \delta_{\varepsilon_{\nu'_j}}^{1 - \prod_{j=1}^k \left(\frac{\varepsilon_{\nu'_j}}{p}\right)^{\mathfrak{W}_j}} \rangle \quad (3)
 \end{aligned}$$

256 The LQSVNWAA operator satisfies the following properties:

257 (i) Idempotency: Let $\delta_j = \langle \delta_{\varepsilon_{\mu'_j}}, \delta_{\varepsilon_{\xi'_j}}, \delta_{\varepsilon_{\zeta'_j}}, \delta_{\varepsilon_{\nu'_j}} \rangle \in \mathbb{N}_{[0,p]}$ ($j = 1, 2, \dots, k$) be a collection of
 258 LQSVNNs where $\delta_1 = \delta_2 = \delta_3 = \dots = \delta_k = \delta$ (say), then $LQSVNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) = \delta$.

259 (ii) Monotonicity: Let $\delta_j = \langle \delta_{\varepsilon_{\mu'_j}}, \delta_{\varepsilon_{\xi'_j}}, \delta_{\varepsilon_{\zeta'_j}}, \delta_{\varepsilon_{\nu'_j}} \rangle \in \mathbb{N}_{[0,p]}$ ($j = 1, 2, \dots, k$) be a collection of
 260 LQSVNNs. If $\delta_j \leq \delta_j^*$ for $j = 1, 2, \dots, k$, then
 261 $LQSVNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) \leq LQSVNWAA(\delta_1^*, \delta_2^*, \delta_3^*, \dots, \delta_k^*)$

262 (iii) Boundedness: Let $\delta_j = \langle \delta_{\varepsilon_{\mu'_j}}, \delta_{\varepsilon_{\xi'_j}}, \delta_{\varepsilon_{\zeta'_j}}, \delta_{\varepsilon_{\nu'_j}} \rangle \in \mathbb{N}_{[0,p]}$ ($j = 1, 2, \dots, k$) be a collection of
 263 LQSVNNs. If there exists $\lambda^- = \langle \min_j(\delta_{\varepsilon_{\mu'_j}}), \min_j(\delta_{\varepsilon_{\xi'_j}}), \max_j(\delta_{\varepsilon_{\zeta'_j}}), \max_j(\delta_{\varepsilon_{\nu'_j}}) \rangle$ and
 264 $\lambda^+ = \langle \max_j(\delta_{\varepsilon_{\mu'_j}}), \max_j(\delta_{\varepsilon_{\xi'_j}}), \min_j(\delta_{\varepsilon_{\zeta'_j}}), \min_j(\delta_{\varepsilon_{\nu'_j}}) \rangle$ then
 265 $\lambda^- \leq LQSVNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) \leq \lambda^+$.

266 **Definition 3.12** Let $\delta_j = \langle \delta_{\varepsilon_{\mu'_j}}, \delta_{\varepsilon_{\xi'_j}}, \delta_{\varepsilon_{\zeta'_j}}, \delta_{\varepsilon_{\nu'_j}} \rangle \in \mathbb{N}_{[0,p]}$ ($j = 1, 2, \dots, k$) be a class of
 267 LQSVNNs, and $\mathfrak{W}_j \in [0,1]$ denotes the weight of δ_j satisfying $\sum_{j=1}^k \mathfrak{W}_j = 1$. Then the

268 linguistic Quadripartitioned single-valued weighted geometric aggregation
 269 (LQSVNWGA) operator is defined as

$$\begin{aligned}
 & LQSVNWAA(\delta_1, \delta_2, \delta_3, \dots, \delta_k) = \prod_{j=1}^k \delta_j^{w_j} \\
 & = \left\langle \delta^{\prod_{j=1}^k \left(\frac{s_{\mu_j}}{p}\right)^{w_j}}, \delta^{\prod_{j=1}^k \left(\frac{s_{\xi_j}}{p}\right)^{w_j}}, \delta^{p - \prod_{j=1}^k \left(1 - \frac{s_{\xi_j}}{p}\right)^{w_j}}, \delta^{p - \prod_{j=1}^k \left(1 - \frac{s_{\nu_j}}{p}\right)^{w_j}} \right\rangle \quad (4)
 \end{aligned}$$

272 The way we have defined the properties of the LQSVNWAA operator, in a similar way
 273 we can define the properties of the LQSVNWAA operator, so it is not repeated here.

274 3.3 Distance measures between two linguistic Quadripartitioned single-valued 275 neutrosophic numbers

276 **Definition 3.13** For any two LQSVNNs $\delta_1 = \langle \delta_{s_{\mu_1}}, \delta_{s_{\xi_1}}, \delta_{s_{\zeta_1}}, \delta_{s_{\nu_1}} \rangle$ and

277 $\delta_2 = \langle \delta_{s_{\mu_2}}, \delta_{s_{\xi_2}}, \delta_{s_{\zeta_2}}, \delta_{s_{\nu_2}} \rangle$ the Hamming distance between them is defined as

$$278 \mathfrak{H}_{\mathfrak{N}}(\delta_1, \delta_2) = \frac{|\delta_{s_{\mu_1}} - \delta_{s_{\mu_2}}| + |\delta_{s_{\xi_1}} - \delta_{s_{\xi_2}}| + |\delta_{s_{\zeta_1}} - \delta_{s_{\zeta_2}}| + |\delta_{s_{\nu_1}} - \delta_{s_{\nu_2}}|}{4} \quad (5)$$

279 **Example 3.14** Let $\delta_1 = \langle s_2, s_0, s_3, s_2 \rangle$ and $\delta_2 = \langle s_3, s_5, s_2, s_3 \rangle$ be two LQSVNNs obtained
 280 from $\mathfrak{N}_{[0,5]}$. Then we obtain their Hamming distance in the following way

$$281 \mathfrak{H}_{\mathfrak{N}}(\delta_1, \delta_2) = \frac{|2-3| + |0-5| + |3-2| + |2-3|}{4} = 2$$

282 **Proposition 3.15** The Hamming distance between two LQSVNNs δ_1 and $\delta_2 \in \mathfrak{N}_{[0,p]}$ is
 283 denoted by $\mathfrak{H}_{\mathfrak{N}}(\delta_1, \delta_2)$ and it satisfies the following properties:

$$284 1. 0 \leq \mathfrak{H}_{\mathfrak{N}}(\delta_1, \delta_2) \leq p$$

285 2. $\delta_1 = \delta_2 \Leftrightarrow \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = 0$

286 3. $\mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = \mathfrak{F}_\mathfrak{h}(\delta_2, \delta_1)$

287 4. If $\delta_1 \leq \delta_2 \leq \delta_3$ for $\delta_3 \in \mathbb{N}_{[0,p]}$ then $\mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) \leq \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_3)$ and $\mathfrak{F}_\mathfrak{h}(\delta_2, \delta_3) \leq \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_3)$.

288 **Proof.**

289 1. We have

$$290 \quad 0 \leq |\delta_{\varepsilon_{\mu'_1}} - \delta_{\varepsilon_{\mu'_2}}| \leq p, \quad 0 \leq |\delta_{\varepsilon_{\zeta'_1}} - \delta_{\varepsilon_{\zeta'_2}}| \leq p, \quad 0 \leq |\delta_{\varepsilon_{\nu'_1}} - \delta_{\varepsilon_{\nu'_2}}| \leq p, \text{ and } 0 \leq |\delta_{\varepsilon_{\nu'_1}} - \delta_{\varepsilon_{\nu'_2}}| \leq p.$$

291 Thus, $0 \leq \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) \leq p$.

292 2. If $\delta_1 = \delta_2$ then it is obvious that

$$293 \quad |\delta_{\varepsilon_{\mu'_1}} - \delta_{\varepsilon_{\mu'_2}}| = |\delta_{\varepsilon_{\zeta'_1}} - \delta_{\varepsilon_{\zeta'_2}}| = |\delta_{\varepsilon_{\nu'_1}} - \delta_{\varepsilon_{\nu'_2}}| = 0$$

294 Which implies $\mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = 0$.

295 On the other hand $\mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = 0 \Rightarrow \delta_{\varepsilon_{\mu'_1}} = \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\zeta'_1}} = \delta_{\varepsilon_{\zeta'_2}}, \delta_{\varepsilon_{\nu'_1}} = \delta_{\varepsilon_{\nu'_2}}$, and $\delta_{\varepsilon_{\nu'_1}} = \delta_{\varepsilon_{\nu'_2}}$ i.e

296 $\delta_1 = \delta_2$.

297 Thus, $\delta_1 = \delta_2 \Leftrightarrow \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = 0$

$$298 \quad 3. \mathfrak{F}_\mathfrak{h}(\delta_1, \delta_2) = \frac{|\delta_{\varepsilon_{\mu'_1}} - \delta_{\varepsilon_{\mu'_2}}| + |\delta_{\varepsilon_{\zeta'_1}} - \delta_{\varepsilon_{\zeta'_2}}| + |\delta_{\varepsilon_{\nu'_1}} - \delta_{\varepsilon_{\nu'_2}}|}{4}$$

$$299 \quad = \frac{|\delta_{\varepsilon_{\mu'_2}} - \delta_{\varepsilon_{\mu'_1}}| + |\delta_{\varepsilon_{\zeta'_2}} - \delta_{\varepsilon_{\zeta'_1}}| + |\delta_{\varepsilon_{\nu'_2}} - \delta_{\varepsilon_{\nu'_1}}|}{4} = \mathfrak{F}_\mathfrak{h}(\delta_2, \delta_1)$$

300 4. Let $\delta_1 = \langle \delta_{\varepsilon_{\mu'_1}}, \delta_{\varepsilon_{\zeta'_1}}, \delta_{\varepsilon_{\nu'_1}} \rangle$, $\delta_2 = \langle \delta_{\varepsilon_{\mu'_2}}, \delta_{\varepsilon_{\zeta'_2}}, \delta_{\varepsilon_{\nu'_2}} \rangle$, and

301 $\delta_3 = \langle \delta_{\varepsilon_{\mu'_3}}, \delta_{\varepsilon_{\zeta'_3}}, \delta_{\varepsilon_{\nu'_3}} \rangle$

302 It is given that $\delta_1 \leq \delta_2 \leq \delta_3$ which implies $\delta_{s_{\mu'_1}} \leq \delta_{s_{\mu'_2}} \leq \delta_{s_{\mu'_3}}, \delta_{s_{\xi'_1}} \leq \delta_{s_{\xi'_2}} \leq \delta_{s_{\xi'_3}},$

303 $\delta_{s_{\zeta'_1}} \geq \delta_{s_{\zeta'_2}} \geq \delta_{s_{\zeta'_3}},$ and $\delta_{s_{\nu'_1}} \geq \delta_{s_{\nu'_2}} \geq \delta_{s_{\nu'_3}}.$

304 Now,

305 $\mathfrak{F}_{\mathfrak{F}}(\delta_1, \delta_2) - \mathfrak{F}_{\mathfrak{F}}(\delta_1, \delta_3) =$

$$\begin{aligned} & \frac{|\delta_{s_{\mu'_1}} - \delta_{s_{\mu'_3}}| + |\delta_{s_{\xi'_1}} - \delta_{s_{\xi'_3}}| + |\delta_{s_{\zeta'_1}} - \delta_{s_{\zeta'_3}}| + |\delta_{s_{\nu'_1}} - \delta_{s_{\nu'_3}}|}{4} \\ & - \frac{|\delta_{s_{\mu'_1}} - \delta_{s_{\mu'_2}}| + |\delta_{s_{\xi'_1}} - \delta_{s_{\xi'_2}}| + |\delta_{s_{\zeta'_1}} - \delta_{s_{\zeta'_2}}| + |\delta_{s_{\nu'_1}} - \delta_{s_{\nu'_2}}|}{4} \\ & = \frac{\delta_{s_{\mu'_3}} - \delta_{s_{\mu'_1}} + \delta_{s_{\xi'_3}} - \delta_{s_{\xi'_1}} + \delta_{s_{\zeta'_3}} - \delta_{s_{\zeta'_1}} + \delta_{s_{\nu'_3}} - \delta_{s_{\nu'_1}}}{4} - \frac{\delta_{s_{\mu'_2}} - \delta_{s_{\mu'_1}} + \delta_{s_{\xi'_2}} - \delta_{s_{\xi'_1}} + \delta_{s_{\zeta'_2}} - \delta_{s_{\zeta'_1}} + \delta_{s_{\nu'_2}} - \delta_{s_{\nu'_1}}}{4} \\ & = \frac{\delta_{s_{\mu'_3}} - \delta_{s_{\mu'_2}} + \delta_{s_{\xi'_3}} - \delta_{s_{\xi'_2}} + \delta_{s_{\zeta'_3}} - \delta_{s_{\zeta'_2}} + \delta_{s_{\nu'_3}} - \delta_{s_{\nu'_2}}}{4} \geq 0 \end{aligned}$$

309 Therefore, $\mathfrak{F}_{\mathfrak{F}}(\delta_1, \delta_2) \leq \mathfrak{F}_{\mathfrak{F}}(\delta_1, \delta_3).$

310 Similarly, we can prove that $\mathfrak{F}_{\mathfrak{F}}(\delta_2, \delta_2) \leq \mathfrak{F}_{\mathfrak{F}}(\delta_1, \delta_2).$

311 **4. Linguistic Quadripartitioned Single-Valued Neutrosophic Soft Sets**
 312 **(LQSVNSSs) and their Properties:**

313 In this part, we first give the notion of LQSVNSSs and then study their various
 314 properties and operations on them.

315 **Definition 4.1** Let \hat{A} be a set of alternatives, \mathfrak{C} be a set of criteria,
 316 and $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ be a continuous finite linguistic term set with cardinality $p + 1.$

317 Then, a LQSVNSS defined on \hat{A} under \mathfrak{C} be denoted and defined as

318 $Y_{\zeta} = \{(\varepsilon_j, \pi_{\zeta}(\varepsilon_j)) : \varepsilon_j \in \zeta, \pi_{\zeta}(\varepsilon_j) \in \Lambda_{[0, \rho]}^A\}$ where $\pi_{\zeta} : \zeta \rightarrow \Lambda_{[0, \rho]}^A$. Furthermore, we represent it in the
 319 following manner:

320
$$Y_{\zeta} = \{(\varepsilon_j, \{(a_i, \langle s_{\mu_{ij}}, s_{\xi_{ij}}, s_{\zeta_{ij}}, s_{\nu_{ij}} \rangle))\} : \varepsilon_j \in \zeta, a_i \in A, \langle s_{\mu_{ij}}, s_{\xi_{ij}}, s_{\zeta_{ij}}, s_{\nu_{ij}} \rangle \in \Lambda_{[0, \rho]}\}$$

321 For simplicity, the set of all LQSVNSSs on \hat{A} under the criteria set ζ and
 322 $\mathfrak{S} = \{s_0, s_1, s_2, \dots, s_p\}$ is denoted by $\Lambda_{[0, \rho]}^{A/\zeta}$.

323 **Example 4.2** Let us assume that $P = \{p_1, p_2, p_3, p_4, p_5\}$ is a set of people whose sputum being
 324 collected by a primary health center for diagnostic purposes whenever the given set of
 325 persons have the symptoms represented by the
 326 set $S = \{s_1 = \text{cough}, s_2 = \text{chest pain}, s_3 = \text{fever}, s_4 = \text{breathing difficulty}\}$.

327 The linguistic term set is

328
$$L = \{l_0 = \text{frequently}, l_1 = \text{always}, l_2 = \text{never}, l_3 = \text{severe}, l_4 = \text{more than one week}, l_5 = \text{last for 6 days}\}$$

329 .The qualitative measurement of expert is represented by the following LQSVNSS:

330 $Y =$

331
$$\{(s_1, \{(p_1, \langle l_1, l_2, l_3, l_1 \rangle), (p_2, \langle l_0, l_4, l_3, l_5 \rangle), (p_3, \langle l_1, l_1, l_3, l_4 \rangle), (p_4, \langle l_1, l_0, l_2, l_1 \rangle), (p_5, \langle l_5, l_5, l_2, l_3 \rangle)\}), (s_2, \{(p_1, \langle l_1, l_0, l_4, l_2 \rangle), (p_2, \langle l_2, l_2, l_3, l_4 \rangle), (p_3, \langle l_3, l_0, l_2, l_4 \rangle), (p_4, \langle l_2, l_2, l_2, l_3 \rangle), (p_5, \langle l_5, l_2, l_2, l_1 \rangle)\}), (s_3, \{(p_1, \langle l_1, l_0, l_2, l_2 \rangle), (p_2, \langle l_4, l_2, l_3, l_5 \rangle), (p_3, \langle l_1, l_2, l_3, l_3 \rangle), (p_4, \langle l_0, l_0, l_0, l_5 \rangle), (p_5, \langle l_4, l_4, l_2, l_1 \rangle)\}), (s_4, \{(p_1, \langle l_5, l_4, l_3, l_1 \rangle), (p_2, \langle l_2, l_2, l_4, l_0 \rangle), (p_3, \langle l_1, l_2, l_0, l_3 \rangle), (p_4, \langle l_3, l_3, l_4, l_5 \rangle), (p_5, \langle l_4, l_4, l_3, l_5 \rangle)\})\}$$

332 For a better understanding the above LQSVNS set can be represented in the following
 333 matrix form given in Table 2 below:

334 **Definition 4.3** Let $Y^1, Y^2 \in \Lambda_{[0, \rho]}^{A/\zeta}$, then Y^1 is an LQSVNS subset of Y^2 if $\Lambda_{ij}^1 \leq \Lambda_{ij}^2$ for all
 335 $i \in \{1, 2, \dots, p\}$ and $j \in \{1, 2, \dots, q\}$ and it is denoted by $Y^1 \subseteq Y^2$.

336 Note: For all $i \in \{1, 2, \dots, p\}$ and $j \in \{1, 2, \dots, q\}$ if $\Lambda_{ij}^1 = \Lambda_{ij}^2$, then equality holds.

337 **Proposition 4.4** Let $Y^1, Y^2, Y^3 \in \Lambda_{[0, \rho]}^{A/C}$, then we have the following:

338 (i) $Y^1 \hat{\subseteq} Y^2$ and $Y^2 \hat{\subseteq} Y^1 \Leftrightarrow Y^1 = Y^2$

339 (ii) $Y^1 \hat{\subseteq} Y^2$ and $Y^2 \hat{\subseteq} Y^3 \Rightarrow Y^1 \hat{\subseteq} Y^3$

340 Proof. The proofs are straight forward.

341 **Definition 4.5** Let $Y_C = \{(\varepsilon_j, \pi_C(\varepsilon_j)) : \varepsilon_j \in C, \pi_C(\varepsilon_j) \in \Lambda_{[0, \rho]}^A\}$ be a LQSVNSS. Then its complement

342 is denoted and defined by $\bar{Y}_C = \{(\varepsilon_j, \widetilde{\pi_C(\varepsilon_j)}) : \varepsilon_j \in C, \widetilde{\pi_C(\varepsilon_j)} \in \Lambda_{[0, \rho]}^A\} = \{(\varepsilon_j, \{(a_i, \pi_{ij})\}) : \varepsilon_j \in C, \pi_{ij} \in \Lambda_{[0, \rho]}^A\}$.

343 **Definition 4.6** Let Y_C^1 and Y_C^2 be two LQSVNSSs. Then their intersection is denoted and

344 defined by

$$345 \quad Y_C^1 \hat{\cap} Y_C^2 = \{(\varepsilon_j, \pi_C^{\cap}(\varepsilon_j)) : \varepsilon_j \in C, \pi_C^{\cap}(\varepsilon_j) \in \Lambda_{[0, \rho]}^A\}$$

$$346 \quad = \{(\varepsilon_j, \{(a_i, \pi_{ij}^{\cap})\}) : \varepsilon_j \in C, \pi_{ij}^{\cap} \in \Lambda_{[0, \rho]}^A\}$$

347 where $\pi_{ij}^{\cap} = Y_{ij}^1 \cap Y_{ij}^2$ for $i=1, 2, \dots, n$ and $j=1, 2, \dots, m$.

348 **Definition 4.7** Let Y_C^1 and Y_C^2 be two LQSVNSSs. Then their union is denoted and

349 defined by

$$350 \quad Y_C^1 \hat{\cup} Y_C^2 = \{(\varepsilon_j, \pi_C^{\cup}(\varepsilon_j)) : \varepsilon_j \in C, \pi_C^{\cup}(\varepsilon_j) \in \Lambda_{[0, \rho]}^A\}$$

$$351 \quad = \{(\varepsilon_j, \{(a_i, \pi_{ij}^{\cup})\}) : \varepsilon_j \in C, \pi_{ij}^{\cup} \in \Lambda_{[0, \rho]}^A\}$$

352 where $\pi_{ij}^{\cup} = Y_{ij}^1 \cup Y_{ij}^2$ for $i=1, 2, \dots, n$ and $j=1, 2, \dots, m$.

353 **5. Applications:**

354 In this section one of the examples of Linguistic Quadripartitioned Single-Valued
355 Neutrosophic Soft Set for various applications is briefly discussed. It is well known that
356 this set is independent of human quantum Turiyam cognition (Singh 2023c). **It**
357 **represents Linguistics and its indeterminacy in four ways as static uncertainty rather**
358 **than dynamic or involvement of Human consciousness.** Let us suppose a problem
359 related to India where some of the students demand teaching in the local language. To
360 solve this issue feedback data is collected. Some of the students agreed for teaching in
361 the local language $\mu_{Q(\xi_i)} \in [0,1]$ denote the truth. Some of the nearby or other state
362 students contradict the statement that why the local language of my state is near hence
363 my state language should be considered $\xi_{Q(\xi_i)} \in [0,1]$. Some of the students from outside
364 India $\zeta_{\mathcal{H}(\xi_i)} \in [0,1]$ who came for a degree wanted to take a degree in any language or
365 were unaware about an issue can be represented via an uncertain degree. The last is
366 educated or intellectual students disagree that teaching locally is harmful to the career.
367 We should go for a global language. It can be represented as falsity membership grades.
368 These types of data sets can be analyzed using the Linguistic Quadripartitioned Single-
369 Valued Neutrosophic Soft Set. Table 3 represents the Tabular representation where x
370 represents the number of students (x_n) and y represents the number of subjects (y_m) and
371 the Quadripartitioned Single-Valued Neutrosophic ($\langle l_1, l_2, l_3, l_4 \rangle$) relations show their
372 feedback for teaching in a local language, its contradiction or uncertainty or rejection.
373 Everyone independently gave their feedback. It will help to analyze the influence of
374 local language in the university, diversity in the university, weak or strong student

375 percentages in the university as well as acceptance of other languages. For more details
376 see Table 3

377 We can also compute the Hamming distance to analyze the similarity among feedback
378 of two departments and their students. In this way, the proposed method helps deal with
379 these types of data sets where contradiction exists between two or more opinions
380 beyond the true or false, or uncertainty. **However, to explore the unknown or impossible**
381 **data that teaching in a multilingual system requires human quantum Turiyam cognition**
382 **as discussed by Singh (2023c).** The authors will try to focus on exploring this area with
383 an illustrative example.

384 **6. Conclusions:**

385 This paper focused on dealing with the uncertainty in qualitative data which are known.
386 To deal with uncertainty in these types of data a Linguistic Quadripartitioned Single-
387 Valued Neutrosophic Soft Sets (LQSVNSSs) and its weighted aggregation operator are
388 introduced. The paper introduces some of the basic concepts and examples for dealing
389 with the education sector data. Shortly, the authors will focus on introducing other
390 metrics for dealing with these types of data using four-valued logic.

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	Fuzzy set	Intuitionistic fuzzy set	Neutrosophic set	Turiyam set	Quadripartitioned
Data	Uncertain	Vague	Indeterminacy	Unknown or impossible	Uncertain and indeterminant
Membership-values	Single-values for true (t) and false (f)	Dependent values for true (t) and false (f)	Independent values for true (t), false (f) and indeterminacy (i)	Independent and dependent values for true (t), false (f), uncertain (i) and liberation (l).	Independent and dependent values for true (t), false (f), uncertain(i), incompleteness (l).
Range	[0, 1]	Dependent: [0, 1]	Independent case: [-3, 3] ⁺ or Dependent case: [0, 1] ⁺	Independent case: [-4, 4] ⁺ or Dependent case: [0, 1] ⁺	Independent case: [-4, 4] ⁺ or Dependent case: [0, 1] ⁺
Undefined objects	No	No	No	Yes	No
Dynamic attribute	No	No	No	Yes	No
Consciousness utilized	No	No	No	Yes	No
Expert to expert	Same representation	Same Representation	Same representation	Varies from expert to expert based on their consciousness	Same representation
Application	Natural Language processing	Pattern recognition	Image Processing, feedback	Robotics, Self-driving cars, cognitive science	Qualitative data measurement

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582 Table 1: The distinction among fuzzy, intuitionistic, neutrosophic, Turiyam and Quadripartitioned

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	S_1	S_2	S_3	S_4	
584	p_1	$\langle l_1, l_2, l_3, l_1 \rangle$	$\langle l_1, l_0, l_4, l_2 \rangle$	$\langle l_1, l_0, l_2, l_2 \rangle$	$\langle l_5, l_4, l_3, l_1 \rangle$
585					
586	p_2	$\langle l_0, l_4, l_3, l_5 \rangle$	$\langle l_2, l_2, l_3, l_4 \rangle$	$\langle l_4, l_2, l_3, l_5 \rangle$	$\langle l_2, l_2, l_4, l_0 \rangle$
587					
588	p_3	$\langle l_1, l_1, l_3, l_4 \rangle$	$\langle l_3, l_0, l_2, l_4 \rangle$	$\langle l_1, l_2, l_3, l_3 \rangle$	$\langle l_1, l_2, l_0, l_3 \rangle$
589					
590	p_4	$\langle l_1, l_0, l_2, l_1 \rangle$	$\langle l_2, l_2, l_2, l_3 \rangle$	$\langle l_0, l_0, l_0, l_5 \rangle$	$\langle l_3, l_3, l_4, l_5 \rangle$
591					
592	p_5	$\langle l_5, l_5, l_2, l_3 \rangle$	$\langle l_5, l_2, l_2, l_1 \rangle$	$\langle l_4, l_4, l_2, l_1 \rangle$	$\langle l_4, l_4, l_3, l_5 \rangle$

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Table 2: Tabular representation of an LQSVNSS

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	y_1	y_2	y_3	...	y_m
596	x_1	$\langle l_1, l_2, l_3, l_1 \rangle$	$\langle l_1, l_0, l_4, l_2 \rangle$	$\langle l_1, l_0, l_2, l_2 \rangle \dots$	$\langle l_5, l_4, l_3, l_1 \rangle$
597					
598	x_2	$\langle l_0, l_4, l_3, l_5 \rangle$	$\langle l_2, l_2, l_3, l_4 \rangle$	$\langle l_4, l_2, l_3, l_5 \rangle \dots$	$\langle l_2, l_2, l_4, l_0 \rangle$
599					
600	x_3	$\langle l_1, l_1, l_3, l_4 \rangle$	$\langle l_3, l_0, l_2, l_4 \rangle$	$\langle l_1, l_2, l_3, l_3 \rangle \dots$	$\langle l_1, l_2, l_0, l_3 \rangle$
601					
602	x_4	$\langle l_1, l_0, l_2, l_1 \rangle$	$\langle l_2, l_2, l_2, l_3 \rangle$	$\langle l_0, l_0, l_0, l_5 \rangle \dots$	$\langle l_3, l_3, l_4, l_5 \rangle$
603				
604					
605	x_n	$\langle l_5, l_5, l_2, l_3 \rangle$	$\langle l_5, l_2, l_2, l_1 \rangle$	$\langle l_4, l_4, l_2, l_1 \rangle \dots$	$\langle l_4, l_4, l_3, l_5 \rangle$

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Table 3: Tabular representation of an LQSVNSS for teaching in Local Language

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